Correspondence

The Resonant Frequency of Interdigital Filter Elements

The purpose of this correspondence is to describe a method of predicting the center frequencies of band-pass interdigital filters.[1] The individual filter elements consist of approximately quarter-wavelength fingers between parallel ground-planes, the fingers being short-circuited at one end and a finite distance from another short-circuit plane at the open end. At present the accepted technique appears to be to separate the short-circuit planes by a quarter-wavelength at the design center frequency and to foreshorten the fingers by an arbitrary amount, based upon the designer's experience. This procedure is rather inexact and tends to shift the center frequency of the filter by an amount which is often unacceptably large, particularly in the case of very narrow-band designs. As an example, the ten percent bandwidth filter of Cristal[2] suffered a shift, when measured, of 3.8 percent from the design center frequency. Therefore, it is apparent that the center frequencies of filters with bandwidths less than three percent could easily be shifted beyond the desired band edges, and although the filter can be tuned by adjusting either the length of the finger or the capacitive gap at the open-circuited end, this method is not always desirable. Tuning mechanisms are not suitable when the filter is for a high power application or when dissipation losses or manufacturing costs must be minimized.

A more precise method of estimating the center frequency can be derived by calculating the resonant frequency of each element, allowing for the capacitances associated with the open end of the finger. These are shown diagrammatically in Fig. 1, where C_f is the fringing capacitance due to the abrupt termination of the finger, and C_p is the "parallel-plate" capacitance to the end wall. Now the input impedance of a short-circuited length (l) of lossless transmission line is given by:

$$Z_{\text{in}} = jZ_0 \tan\left(\frac{\omega l}{c}\right)$$

where (Z_0) is the characteristic impedance of the line, (c) is the velocity of light in air, and (ω) is the angular frequency. Therefore, the finger will be resonant when

$$jZ_0 \tan\left(\frac{\omega_0 l}{c}\right) + \frac{1}{j\omega_0 C_t} = 0 \qquad (1)$$

where C_t is the total "end" capacitance $(C_f + C_p)$. As far as is known, an exact expression for C_f, in slab line, does not exist, but an approximation can be obtained from Whinnery et al.[3] This is shown graphically in Fig. 2.

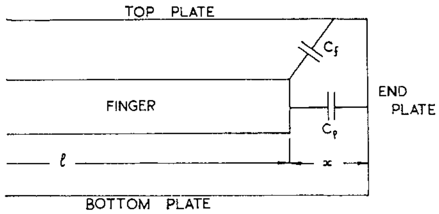

Fig. 1. Capacitances associated with interdigital filter elements.

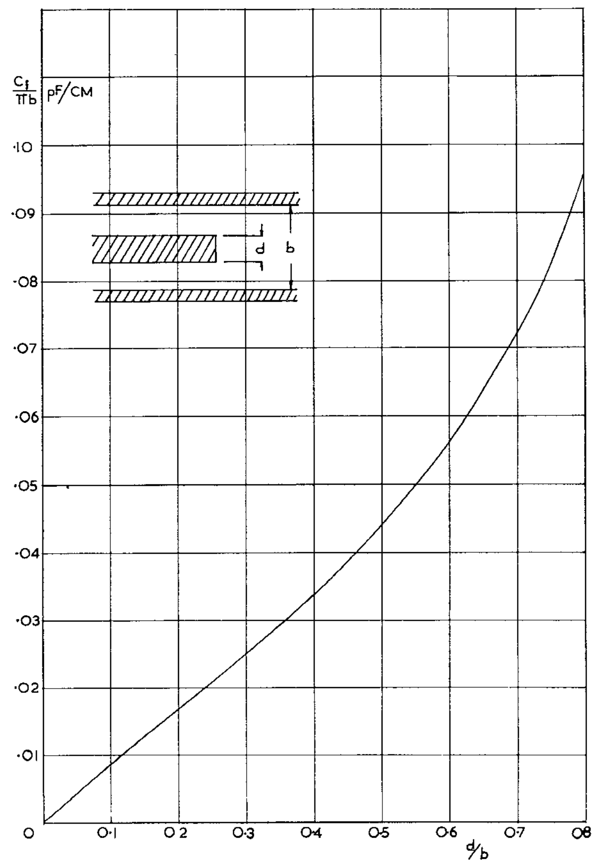

Fig. 2. Discontinuity capacitance (C_f) for open-circuited line.

The dimension b' represents the "equivalent" coaxial outer diameter, which may be obtained by the conformal transformation $\omega = \tan z$. This mapping transforms a slab-line configuration with a nearly circular center conductor in the z plane into a coaxial section in the ω plane. The diameter of the inner conductor remains unchanged for typical filter dimensions, but the "equivalent" outer diameter will be given by

$$b' = \frac{4b}{\pi}$$

where b is the ground plane separation of the slab line.

The "parallel-plate" capacitance (C_p) for air dielectric is given approximately by the expression

$$C_p = \frac{0.0885\pi d^2}{4x}. \qquad (2)$$

The units will be pF when the dimensions as shown in Figs. 1 and 2 are in centimeters.

This method of arriving at C_t is rather approximate, but due to the proximity of the short-circuit plane, C_f will tend to be overestimated; whereas, due to fringing effects, C_p will always be underestimated. Thus, to some extent C_t is self-compensating and, for dimensions commonly used in interdigital filter designs, provides a reasonable estimation of the capacitive end-loading.

The characteristic impedance Z_0 in (1) will depend upon the scaling factor used in the design equations, but is usually chosen for optimum unloaded Q, which is assumed to occur when $Z_0 = 76$ ohms. Knowing Z_0, if ω_0 and x can be specified, the filter may be designed by solving (1) for l.

The above procedure was adopted in designing a three-section 0.1-dB ripple

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[1] G. L. Matthaei, "Interdigital band-pass filters," *IEEE Transactions on Microwave Theory and Techniques*, vol. MTT-10, pp. 479–491, November 1962.

[2] E. G. Cristal, "Coupled circular cylindrical rods between parallel ground planes," *IEEE Transactions on Microwave Theory and Techniques*, vol. MTT-12, pp. 428–439, July 1964.

[3] J. R. Whinnery, H. W. Jamieson, and T. E. Robbins, "Coaxial-line discontinuities," *Proc. IRE*, vol. 32, pp. 695–709, November 1944.

Tchebychev filter with a bandwidth of 20 Mc/s centered upon 6.900 Gc/s. The ground plane spacing b was chosen to be 0.5 inch, which resulted in a finger diameter d of 0.199 inch, and the spacing x was chosen to be 0.125 inch. From Fig. 2, the fringing capacitance (C_f) was found to be 0.132 pF, and from (2) the "parallel plate" capacitance (C_p) was 0.055 pF, giving a total end capacitance (C_t) of 0.187 pF.

By rearranging (1),

$$l = \frac{c}{\omega_0} \tan^{-1}\left(\frac{1}{\omega_0 Z_0 C_t}\right) \tag{3}$$

and, hence, the finger length could be calculated. The measured center frequency of this filter was found to be 6 896 Gc/s, whereas scaling from the design adopted by Matthaei[1] and Cristal[2] resulted in a filter having a center frequency of 7.00 Gc/s. A number of existing filters in the frequency range 500 Mc/s to 7 Gc/s have been analyzed by graphically solving (1), which is a transcendental equation in ω, for each mechanical structure. The center frequencies calculated by this method would have reduced the discrepancies between design and measurement by at least 70 percent in all cases. As an example, the filter quoted by Cristal[2] had a design center frequency of 1.500 Gc/s. By applying the above procedure, a center frequency of 1.543 Gc/s would be predicted, which is in close agreement with the measured center frequency 1.557 Gc/s.

It is interesting to note that to obtain a particular resonant frequency for a given end plate separation, the length of each finger in a filter is a function of the diameter. Since the fingers in the center of these filters are often of equal diameter, they will all have the same resonant frequency. However, the fingers at the ends of the filter are usually of different diameters, and ideally the finger lengths should be adjusted accordingly. This adjustment is not usually made and provides an alternative explanation of the slight adjustments in coupling (and effectively Z_0) which always seem to be necessary between the end elements.

In conclusion, it is felt that, whereas a more rigorous analysis of these discontinuities would undoubtedly lead to even closer agreement, the above procedure should prove useful in reducing considerably the gap between the design and measured center frequencies of interdigital filters.

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Application of the Point-Matching Method in Waveguide Problems

The determination of cutoff frequencies of waveguides with arbitrary cross section by the point-matching technique was recently studied by Yee and Audeh [1], [2].

It is interesting to point out that the point-matching technique or collocation method is a well-known and popular procedure in several areas of applied mechanics. It has been used in boundary value and eigenvalue problems [3]–[11]. The problem treated by Baltrukonis [9] is governed by the same differential system which governs the propagation of electromagnetic waves in hollow-piped waveguides. Baltrukonis [9] deals with a star-shaped boundary given by the equation

$$S(r, \theta) \equiv r - (a + b \cos 4\theta) = 0.$$

The circle is one curve of the family. The first four eigenvalues were calculated and plotted in function of the dimensionless parameter b/a. When b/a approaches zero the boundary is circular, for which the exact solution is known. This study shows that, for the problem under consideration, the calculated eigenvalues depend rather drastically on the distribution of points. Furthermore, little or no convergence is demonstrated for as many as seven collocation points taken within an octant of the boundary.

Jain [10] has introduced a new criteria for the collocation procedure. In this procedure one requires that the error at adjacent matching points be equal in magnitude but opposite in sign. Furthermore, the error at the matching points must be larger than that at any other point. This technique seems to yield better results than the straight collocation method [11].

One of the main advantages of the point matching is its simplicity. On the other hand, it should be emphasized that many uncertainties exist regarding accuracy of the results when the method is applied to new problems.

Indication of convergence can usually be obtained by using conformal mapping along with various approximation techniques [12]–[14]. One of the main advantages of using conformal mapping is that some bounding techniques can generally be used once the boundary conditions are identically satisfied [13]–[15]. It was shown [14] that the use of conformal mapping and Galerkin's Method leads to excellent results.

In summary, it is the opinion of the author that additional study of all these approximate techniques from a unified point of view is needed.

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Resonances of a Cylindrical Cavity in a Lossy Compressible Plasma

Wait [1] has derived equations describing the resonances of a cylindrical cavity in an isotropic, lossy compressible plasma. The purpose of this communication is to consider low-frequency resonances when the effects of losses and compressibility are small.

Consider a cylindrical cavity which is a free space region immersed in an isotropic compressible lossy plasma. The plasma has permeability equal to the free space value μ_0 and permittivity ϵ where

$$\frac{\epsilon}{\epsilon_0} = 1 + \frac{\omega_N^2}{(\nu + i\omega)i\omega} \cdot \tag{1}$$

In (1), ϵ_0 is the permittivity of free space, ω_N is the angular plasma frequency of the electrons, and ν is the electron collision frequency. The fields have the time factor $e^{i\omega t}$, where ω is the angular frequency and t is the time. Let [1]

$$\tau_p^2 = -\frac{\omega^2}{u^2}\left(1 + \frac{\nu}{i\omega}\right)\frac{\epsilon}{\epsilon_0} \tag{2}$$

where u is the speed of electron acoustic waves in the plasma.

Suppose that the dimensions of the cavity are small compared with the electromagnetic wavelength both in free space and in the plasma. That is, $|\tau a| \ll 1$ and $|\tau_e a| \ll 1$, where $\tau^2 = -\omega^2\mu_0\epsilon_0$, $\tau_e^2 = -\omega^2\mu_0\epsilon$, and a is the radius of the cavity. Then the resonance condition is [1]

$$\frac{\epsilon}{\epsilon_0} + (1 - \delta_n) = 0 \tag{3}$$

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